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SMALL-SCALE FOOTING STUDIES:
A REVIEW OF THE LITERATURE

Appendix B

for

Preliminary Design Study for a Dynamic Soil Testing Laboratory

by

J. E. Roberts

Massachusetts Institute of Technology
Department of Civil and Sanitary Engineering
Soil Engineering Division

July 1961

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July 1961

Research Directorate
AIR FORCE SPECIAL WEAPONS CENTER
Air Force Systems Command
Kirtland Air Force Base
New Mexico

Approved:

for LEONARD A. EDDY
Colonel USAF
Director, Research Directorate

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ABSTRACT

A review is made of the literature on small-scale testing of footings on soil, particularly of efforts to determine the static ultimate bearing capacity. Literature published earlier than 1960 is discussed, while later reports are only listed in the bibliography. The review was performed as a part of a preliminary design study for a dynamic soil testing laboratory, and was intended to determine why some past efforts were worthwhile and others essentially worthless. It is concluded that the best success is achieved in nonquantitative verification of the modes of deformation and patterns of behavior, particularly of the size, shape, and presence of rupture zones. Good results are obtained from attempts to supplement established theories with empirical correction factors, but attempts to verify quantitative relationships or to establish numerical values have generally failed.

PUBLICATION REVIEW

This report has been reviewed and is approved.

John J. Dishuck

JOHN J. DISHUCK
Colonel USAF
Deputy Chief of Staff for Operations

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PREFACE

The work reported herein was carried out under Contract No. AF29(601)-1947, Preliminary Design Study for a Dynamic Soil Testing Laboratory, and this document constitutes part of the final report in fulfillment of the contract.

A major task of the contemplated laboratory would be performance of small-scale tests involving footings or buried structures subjected to dynamic loadings. Situations of this type fall into the general category of problems in soil-structure interaction. In designing the experimental facilities of such a laboratory, it obviously is desirable to study previous attempts at small-scale soil-structure interaction experiments. One important observation in this connection is that small-scale experiments are regarded with considerable skepticism by many eminent foundation engineers, especially those within the United States. Yet many such experimental programs have been carried out, especially in the areas of earth pressure on retaining walls and the ultimate bearing capacity of footings subjected to static loads, and at least some of these programs have made valuable contributions to our store of knowledge. Thus a main objective in this review of previous small-scale testing is to discern why certain programs achieved their objectives and were deemed to be successful and worthwhile, and why other extensive experimental programs have been held to be essentially worthless.

One particular area of small-scale testing was selected for intensive study: the determination of the static ultimate bearing capacity. Many small-scale footing tests have been conducted in a number of laboratories, and so there is extensive literature to be reviewed. Moreover, there is considerable interest in the dynamic counterpart of such tests. Because the results of this study are of general interest to the Civil Engineering profession, this appendix has been published separately from the remainder of the final report.

This study is the work of Professor James E. Roberts of the Soil Engineering Division, Department of Civil and Sanitary Engineering*. Dr. T. W. Lambe, as

*Prof. Roberts is no longer at the Institute

Head of the Soil Engineering Division, has over-all responsibility for the research work. All work on the research contract has been under the immediate supervision of Prof. Robert V. Whitman.

The literature review and preparation of the written material in this appendix were performed during the summer and fall of 1959, and the written material has received only editorial change since that time. Hence, results of some recent small-scale testing of footings, notably at Armour Research Foundation and the Waterways Experiment Station, have not been discussed in the report, although listed in the Bibliography.

B.1 INTRODUCTION

This appendix contains a review of the available literature pertaining to small-scale or model tests involving footings. To be more specific, the review is limited, for the most part, to small-scale experiments aimed at the study of failure by rupturing of the soil beneath the footing. Thus, the soil property of greatest interest is the shear strength. For tests involving dry or saturated sands, the strength of the soil is properly described by the inclination φ of the Mohr rupture line, and φ will generally be referred to as the friction angle. The strength of clay soils is discussed in terms of the unconsolidated undrained shear strength (one-half of the maximum deviator stress) of the clay for the condition in which the soil was placed in the test apparatus.

Today there exists a reasonably complete and accurate theory for calculation of the rupture load for the case where a long strip footing rests on the surface of soil or at shallow depths below the surface: LUNDGREN and MORTENSEN (1953) and SOKOLOVSKI (1960). Even so, the implications of this theory have not been reduced to a form in which they are readily useable. The first satisfactory theory (approximate but very satisfactory) was published by TERZAGHI (1948). Terzaghi introduced the bearing capacity factors N_c, N_γ, and N_q. The use of these factors is commonplace today, and they will be used to express many of the results presented in this appendix. MEYERHOF (1950, 1951) extended the use of the approximate methods suggested by Terzaghi.

It is important to realize the general limitations of the approximate calculation methods developed by Terzaghi and subsequent workers who followed his lead. These same limitations apply to the more complete theory of Lundgren-Mortensen and Sokolovski. The most important limitations are: (1) the calculation methods apply only to long strip footings, and (2) the calculation methods apply only to soils which fit the description of pure plastic; i.e., no significant strains occur until rupture occurs. (It might be added that the calculation methods can be used in an accurate way only when the pore pressures existing at each point within the soils mass are known or where the $\varphi = 0$ principle applies.)

The various experimental programs reported herein appear to have had various objectives, depending upon the time at which they were conducted and the view-point of the researcher. The most common objectives were:

(a) Development of numerical values of bearing capacity for use in design work.

(b) Development of empirical rules for extrapolating the results of small-scale tests.

(c) Investigation of general mechanism of rupture below loaded areas.

(d) Development of empirical correction factors for Terzaghi's theory to permit use of theory with other than long strip footings, with other than shallow or surface footings, and with soils in which deformations occurring before complete rupture are important.

The reader should judge for himself which types of objectives were successfully achieved through use of small-scale tests.

B.2 SUMMARY OF SMALL-SCALE FOOTING TESTS ON SAND

Twenty-one summaries of small-scale tests on sand, dating from 1930 to 1958, are presented chronologically in this section. Pertinent data presented in the summaries, relating to bin size and shape of loading area, grain size, distribution of the sand, and techniques of soil placement and control, where presented, are summarized in Table I.

B.21. Fellenius (1930)

The earliest detailed study of footing behavior reviewed is that of Fellenius. Footings of different dimensions (5 to 30 cm wide and 5 to 6 cm long) were tested to study the effect of size and shape on bearing capacity (that capacity at which the plate continued to sink without further increment of load), and the results were compared with theoretical analyses using a circular arc failure surface. The major conclusion was that the bearing capacity of rectangular or elliptically loaded areas on sand is proportional to the least width.

Unfortunately, no details were given concerning the method of placing the sand, and apparently there was no determination of the sand's density. In addition, the friction angle was merely stated to be about 31°, and no test data were given. Subsequent studies of the extent of the failure zone led MEYERHOF (1950) to feel that the tank used by Fellenius may have been too small for all but the 5 cm and possibly 10 cm wide areas.

Fellenius's solutions based on a circular arc rupture surface agree reasonably well with Terzaghi's ($\varphi = 31^\circ$, $N_\gamma = 25$; $N_q = 22\text{-}1/2$); however Fellenius points out that his solution is for the case of zero friction between footing and soil and that this friction should increase the bearing capacity.

B.2.2 Jandris and Riley (1930)

During thesis research at M.I.T., Jandris and Riley studied the effect of the shape of the loaded area on bearing capacity for a single area of sixty four (64) sq.cm. in circular, square and octagonal shapes. One test at each of three void ratios was performed for each shape. By interpolating the results, the bearing capacity (that pressure at which the settlement equaled 0.4 inches) of the different shapes were compared at a common void ratio of 0.6 as follows:

Shape of Footing	Diameter	Bearing Capacity
circle	9.03 cm	0.84 kg per sq cm
square	8.0 cm	0.79 kg per sq cm
octagon	8.8 cm	0.74 kg per sq cm

No shear strength data were obtained, and there is insufficient data from which to draw any general conclusions regarding influence of shape.

B.2.3 Gilboy (1930)

Tests were performed on clean dry Plum Island sand in three states of compaction (dense, medium and loose) using three concrete octagonal footings with areas 1, 3 and 6 sq. feet in a concrete bin 14 x 14 ft. filled to a depth of 7 feet. Densities were measured after the tests using long brass tubes, 5 inches long and 3 inches in diameter, for sampling. The test results showed that at pressures small with respect to the ultimate pressure, the stress required to produce a given settlement is practically independent of the footings size. For the tests on dense sand the loads available were insufficient to cause failure of the larger footings, and for the tests on loose sands, settlements were so excessive under small loads that true failure could not be determined. In connection with these tests it was found that a 2-3/4 inch 30° cone penetration device gave a static penetration resistance approximately equal to the load on 1 sq.ft. required to produce 1 cm settlement of the footing. The variation of displacement of the soil mass with depth was also determined, and it was found that vertical displacements disappear at depths greater than 1-3/4 footing diameters.

B.2.4 Kogler (1933)

In a discussion of a paper by GILBOY (1933), Kogler presented the results of a series of tests performed in Freiburg, Germany. Seven different circular areas with diameters ranging from 8 cm to 100 cm were tested to determine the relation between the settlement and diameter of a loaded area. The main conclusion drawn from the test data (Fig. 1) is that the settlement of a loaded area of sand having equal average contact stresses increases with the increasing diameter of the area although not necessarily in proportion to it.

B.2.5 Pandya (1931, 1933)

A very detailed series of tests were performed between 1931 and 1933 at M.I.T. by Pandya to study the effect of width and depth on the settlement and bearing capacity of a fine uniform sand which was placed at a single average density ($e = 0.66$; relative density = 0.74). Shear box tests showed a very small change in friction angle with density (29.5° loose to 31.5° dense) at pressures above 0.7 kg/cm^2. Although no tests were performed at lower pressures, the experimental curves at higher pressures, when extrapolated pass through the origin. The influence of size on bearing capacity is shown in Figure 2. Although the data are not conclusive, it appears that the bearing capacity of surface footings is a function of the least width and is essentially independent of footing shape. However, the bearing capacity of a strip footing increased with depth at a slower rate than did that of a square footing. The test results showed that increasing the depth of a foundation greatly increases the bearing capacity and noticeably reduces the settlement under a given load (the bearing capacity increase was found to be proportional to $(^D/_B)^Z$ where Z is somewhat less than unity for the range of depth tests). The settlement data on surface footings (Figure 3) tend to confirm Gilboy's conclusion that for a given stress not too near the ultimate, the settlement is practically independent of the size of the footing. The relation between settlement and depth for a wall footing is similar to that of the square, but the decrease of settlement with depth is less pronounced.

B.2.6 Golder (1941)

In this study, footings, three inches and six inches wide with lengths up to 30 inches, were tested in a wooden box 3' 6" x 3' 6" filled with 14 inches of dense, dry, coarse sand in order to determine the effect of footing shape. Golder concluded from the results that bearing capacity is a function of width and not of shape. And, for the dense sand, the rate of increase of bearing capacity decreases as the width of the footing increases. Only three tests, however, were run on 3 inch wide footings because the load apparatus was not suitable for small loads.

Two cylindrical metal containers with open tops were placed in the sand to determine the sand density and were removed after testing. The sand density was found to vary between 108 and 112 per cu.ft. with a mean value of 110.5. Shear box tests yielded friction angles between 39 and 48 degrees for dense sand depending on the pressure.

B.2.7 Bekker (1948)

Wooden footings were tested in a glass front box in order to photograph and observe the failure zone. Unfortunately, neither details of the test set-up nor test data are given. According to Bekker, the wooden footings were able to support almost twice the load computed on the basis of Terzaghi's solution. The photographs revealed, Bekker stated, that the boundary of the elastic wedge rose at an angle of 60^o to the horizontal, and that, by using this wedge, agreement was effected between computed and measured bearing capacity for sand with a friction angle of 35^o.

B.2.8 Peynircioglu (1948)

The mechanism of shear failure (i.e. extent and shape of the sliding surfaces) was studied by taking time exposure photographs of the soil mass during loading of various shaped footings in a glass tank, 55 x 33 x 26 cm. For other than strip footings, one half of a footing was tested adjacent to the glass wall which became a plane of symmetry.

For footings on a horizontal surface, the photographs indicated: (a) that the curved part of the sliding surface was practically a logarithmic spiral, (b) in the early stages of the test, sand particles moved almost symmetrically relative to a vertical axis of symmetry, (c) as soon as a definite sliding wedge formed, the footing load dropped to a minimum and remained almost constant during further settlement, (d) in all tests failure occurred as a sliding to one side. For footings on a level surface the sliding surface extended outward a distance of three times the width and extended to a depth equal to three-fourths of the width. Although no load test data were given, the usefulness of such a photographic technique in model studies is clearly demonstrated.

B.2.9 Meyerhof (1948,1950,1951,1953,1955)

In work related to this series of experiments Meyerhof extended the plastic theory analysis for surface footings to shallow and deep foundations and the solution to eccentric and inclined loads. The test programs served, then, as a check on these theoretical approaches.

Most of the tasks were conducted in a stiffened steel tank 18 in. x 15 in. x 18 in. A galvanized tank, 30 in. x 20 in. x 24 in., was used to study the effect of groundwater conditions. In addition, a wooden box with a glass front, the inside of which could be smoked, was used to observe soil movements. The soil used for the sand tests was a clean, angular, uniform sand having a size range 0.2 mm to 0.6 mm with 50% finer than 0.4 mm. Three density states were used: dense (n value not indicated), medium (n = 37%) and loose (n = 45%). Shear box tests on air dry soil (w = 0.2%) showed the maximum friction angle to be a function of both initial porosity and pressure, but the final friction angle (31°) was independent of both factors. Tests on saturated sand gave a maximum angle for dense sand 2° higher than that for the dry soil and for loose sand 1° higher. The final angle for saturated sand was slightly larger (31-1/2°). The density of the sand in the bin was checked by 3" diam. x 1-1/2" open tins which were buried at different depths.

The footings (circular, square, and rectangular) were brass and had a maximum width of 1 inch. The load was applied through a proving ring at a very slow rate (0.02 to 0.04 in. per minute). This type of loading made it possible to determine the load deflection behavior after failure. The small size of the footing was dictated by the size of the box.

In a discussion of Eastwood's paper (1951) Meyerhof points out that according to preliminary studies on dense sand, a clear distance of ten times the width is needed at the sides of rectangular footings, and a clear distance of five times the width is needed both at the ends of rectangular footings and at the sides of square or circular footings. In addition, a clear depth of at least five times the width is needed for any shape footing. This recommendation was adopted because movements outside the failure surface, which were generally radial, could be observed to a distance of 2 to 3 times the dimensions of the failure surface.

The tests for concentrically loaded footings on different density sands are described by Meyerhof as follows:

> Dense Sand: The load settlement curve is similar to the stress-strain curve in that the load reaches a maximum and then with further settlement drops back to a smaller final value. At the maximum point, failure surfaces show up at ground surface. As the depth of footing increases, the difference between the maximum value and the final value decreases. Generally, the footings (especially at shallow depths) tilted to one side, and the sand surface rose uniformly and abruptly for shallow footings. After the soil had been sheared, it was probed with wire, and a marked change in resistance between the disturbed and undisturbed zones was noted.
>
> Compacted Sand: As in the case of the tests on dense sand, the load settlement curves were similar to the stress-strain-shear curves. The size of the failure zone proved to be practically independent of foundation depth and was governed by the compressibility of the sand. For compacted sand a difference between disturbed and undisturbed zones could not be detected by probing as in the previous tests.

Loose Sand: Local shear failure occurred in all cases. Upon loading, settlement occurred at an increasing rate until the bearing capacity was reached. Beyond this the stress required for further settlement was almost directly proportional to settlement. The bearing capacity was governed by compressibility, and failure did not spread to the surface even with surface footings. For surface footings, the ground surface developed a shallow trough and only showed a slight rise at some distance for strip footing. Timed photographs, using the smoked glass, indicated that ground movements were confined to the immediate vicinity of the foundation, and, near the base, movement was almost entirely vertically downward and the size of the failure zone was independent of depth.

The important factors determining the bearing capacity which were mentioned are:

(a) Foundation: size, shape, roughness, depth, and manner of placing

(b) Soil: stress prior to loading, strength and deformation characteristics

In summary, the following conclusions were drawn from the results of the tests:

(a) Bearing capacity increases with the foundation width at a decreasing rate which is considered to be a result of the smaller friction angle at larger pressures on the failure surface (more noticeable for dense sand and for the small footings used).

(b) Bearing capacity increases in almost direct proportion with footing depth within range investigated.

(c) The bearing capacity of square footings is less than that of rectangular footings, especially at the surface on dense sand where the maximum bearing capacity was about 30% less.

(d) For footings on loose sand the bearing capacity is independent of shape.

(e) The bearing capacity of footings with a smooth contact surface is 1/2 that of a rough footing.

(f) Bearing capacity is directly proportional to the unit weight and is influenced by ground water conditions directly as they influence the effective unit weight.

(g) For eccentrically loaded footings, small scale footing tests indicated that a simple solution is obtained by assuming that the bearing capacity is identical to that of a centrally loaded footing if:

$$B^1 = B - 2e$$

Where B = actual width
e = eccentricity
B^1 = effective reduced width

The results of the experimental shallow foundations tests showed reasonable agreement with theory. The theoretical approach for circular footings was only tentative, and Meyerhof suggests an empirical shape correction factor. However, for shallow foundations in loose sand and deep foundations in other sands, compressibility is increasingly important. A rigorous theory in which this factor was taken into account proved to be unsuccessful, and therefore an empirical compressibility factor was introduced by which the real friction angle was reduced.

Shear strength is the most important soil property, and the effects of porosity and pressure must be considered. MEYERHOF (1948) indicated satisfactory agreement between theory and experiment for surface footings when the friction angle was determined for the average pressure acting along the failure surface (approximately 1/10 of the applied surface stress).

B.2.10 Davis and Woodward (1949)

In connection with a graduate course in soil mechanics, rectangular footings, 1 to 1-1/2 inches wide with lengths of 10 to 24 inches, and a circular footing, 12 inches in diameter, were tested in a tank of such dimensions that at the surface of the soil its dimensions were from 1-1/2 to 3 ft., and all depths were at least eight times the footing width. Soil movements were studied by use of glass beads, layers of finely ground white silica (it is noted that the soil in these tests was moistened) and an X-ray technique used to trace the displacement of lead shot. In the latter tests, however, the thickness of the container was limited to 4-1/2 inches.

The studies indicated: (a) displacements are negligible at depths below 3 footing widths, (b) the failure zone, when developing on one side, does not extend outward beyond 5 widths, (c) lateral deformation is inappreciable beyond 6 footing widths and (d) the failure surface closely follows the shape of a logarithmic spiral. With the sandy soil used (nothing more definitive was given) direct shear and triaxial tests indicated a friction angle of the order of 36^{o} for medium to high density and moderate confining pressure. It was stated that loose soil tests at very small pressures gave a friction angle as low as 15 degrees. The authors claim that when the friction, which could be mobilized along the failure arc due to the stresses generated by the footing load itself, was considered, then reasonable agreement was obtained between the computed and observed bearing capacities.

Unfortunately, the details of this analysis are not given, and therefore, this summary cannot be further clarified.

B.2.11 Eastwood (1951)

Footings 3, 4, and 6 inches wide and from 6 to 30 inches long, were tested on a dense, medium sand which was placed in a wooden box 4' o" x 3' 6" x 16". However, Eastwood states that the box was too small for the six inch wide footing, and therefore, only the results of the tests on the 3 and 4 inch wide footings were significant. Shear box tests indicated a friction angle equal to 42-3/4^{o} which was virtually constant for all pressures up to 1.5 tons per sq.ft. Density was checked by means of open topped cans which were dug out after the test. Load was applied through a proving ring by a hand operated screw jack. During a test the load reached a maximum and then dropped from 50% to 80% of its maximum value. A study of the extent of the shear zone was made by placing strips of tissue paper at the edge of the soil mass and observing them at the conclusion of the test.

Most rectangular footings failed with a distinct ultimate load accompanied by slip to one side; the load settlement curve for square or rectangular footings simply passed into a steep and fairly straight tangent for a time after which the load remained constant or slightly increased.

A comparison of the experimental results with the Terzaghi and Prandtl formulae gave reasonable agreement, but the rate of increase of bearing capacity appeared to decrease with increase in footing width. The results agreed with Golder's results that the bearing capacity of rectangular strip footings is independent of length.

Tests on inundated footings showed less than a 20 percent reduction in bearing capacity compared to the tests on dry sand. This is a considerably smaller reduction than generally expected, and Eastwood attempted to explain this by noting that the observed failure mechanism does not agree with those assumed by Krey, Prandtl or Terzaghi. However, in a discussion of Eastwood's paper, Meyerhof suggested that the results might be due to apparent cohesion since at failure there was an insufficient cover of water at ground level. Golder also questioned Eastwood's results regarding inundation and presented data of his own which showed the bearing capacity to be directly proportional to effective unit weights.

B.2.12 <u>Bresner and Wright (1953)</u>

Plates 24 inches long and from 1.87 to 14.8 inches wide, were tested on a compacted sand (density about 110 lbs.per sq.ft.) to study the effect of width on the subgrade modulus of sand. The size of the bin used appears to be about 6 feet long and about 2-1/2 feet wide (no indication of the depth is given). According to the authors, stresses and deflections varied linearly with applied load. The results showed that the subgrade modulus was very high for narrow plates and fell off rapidly with increasing width. However, the authors indicate that above a width of 15 inches for rectangular plates and 24 inches for circular plates the modulus would reach essentially a constant value. These results, as pointed out by CHAPLIN (Geotechnique V 4, p. 42), are of limited value and cannot be considered as generally valid because of the relatively small test bin and the absence of tests at additional densities, but they do suggest that settlement of footings on sand is a function of the width of footings.

B.2.13 Sywestrowicz (1953)

A series of tests was performed in a glass walled container to observe the displacement field in tamped sand during indentation by a 2 inch wide punch.

In each test a maximum load was obtained at which point two symmetrical slip surfaces started to form. As the load dropped off, movement became concentrated along one of the slip surfaces. Eventually movement continued along this surface with approximately constant load. It was possible to detect the movement of individual grains of sand, and deformation during loading could be studied. By measuring the changes of grid areas and observing the characteristics of deformation it appeared that the main volume change occurrred in the discontinuity layer. These tests were of a preliminary nature and no detailed study of shear strength or adequate control of density was attempted.

B.2.14 Landale (1954)*

As part of a study of the behavior of footings under dynamic load, tests were conducted on 3 inch and 6 inch square footings in a concrete bin 5' x 5' x 4' deep. The soil was tamped or compacted at a water content varying from 3-1/2 to 6 percent in 3 inch layers to a void ratio of approximately 0.75 (loose) or approximately 0.64 (dense). Direct shear and triaxial compression tests yielded a range of friction angles varying from 34 to 44 degrees, depending on the density and pressure.

The static load tests on moist, dense sand gave the same bearing capacity for both the 3 inch and 6 inch footings. For the 3 inch footing, the value was about 5 times the Terzaghi value for a friction angle of 44°.

Subsequent investigations (stability studies of vertical cuts in the sand) indicated that the moisture present was developing by capillarity a maximum apparent cohesion of about 41 lbs. per sq.ft. When the effect of this apparent cohesion was taken into consideration a reasonable agreement between the tests results and Terzaghi's formula was obtained.

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*These results are described in less detail in M.I.T. (1954)

B.2.15 Jumikus (1956)

The mechanism of failure of sand under oblique loads was studied by placing in a glass walled tank a fine dry sand having a friction angle of 30 degrees (no data was given on the influence of density and pressure). Failure surfaces were detected by making a grid of black stained and layers 3 mm thick and 100 mm apart and photographing the mass after rupture. In all of the photographs shown, clearly defined failure zones were evident, and the surface of rupture was symmetrical with a logarithmic spiral shape.

B.2.16 Bond (1956)

This work deserves special attention. because it is one of the few investigations to be concerned with the problem of developing and experimentally checking a quantitative approach to the problem of predicting the settlement of footings on sand. Bond adopted the appraoch of ROWE (1954) who hypothesized that "... the solutions to the states of stress within a soil mass, and adjacent to a rigid boundary, hitherto applied to failure problems using maximum values of friction angles, Φ and Δ, apply also where the soil is not failing and where the friction angles are φ and δ." (φ and δ, in this case, refer to developed friction angles.)

Bond refers to his approach as the "center line strain" method and limits his study to circular footings because of the axial symmetry. Along the vertical axis of the footing the major principal stress σ_1 is vertical, and the intermediate and minor principal stresses, σ_2 and σ_3, which are equal, are horizontal. The relationship between σ_1 and σ_3 along the axis can be represented by the expression:

$$\sigma_1 = \sigma_3 \tan^2 \left(\frac{\pi}{4} + \frac{\varphi}{2}\right)$$

where φ is the angle of maximum obliquity (or the developed friction angle).

For a given foundation stress, the values of vertical stress on the center line are obtained from Boussinesq's equation. The value of φ is obtained by considering the footing to be supported by an ideally plastic rigid material; i.e., it is assumed that the existing theories for determining bearing capacity can also be used to determine the developed φ under the given contact stress with σ_1 and φ. With σ_1 and φ determined, σ_3 can be evaluated from the expression above. With the value of σ_1 and σ_3 evaluated for various points along the center line, the corresponding vertical strain at each point can be determined from the results of triaxial compression tests on soil at the same initial density. The strain can then be integrated to determine the surface settlement.

Tests were conducted on circular foundations with diameters varying from 3-1/2 inches to 12 inches placed at depths varying from 0 to 48 inches in a steel tank 8' x 4' x 4'. Densities were evaluated with a calibrated static penetrometer two inches in diameter. In addition to studies of the foundation settlement, the distribution of vertical strains on the center line of the footing was measured by the technique of burying an anchor plate at various depths below the center of the footing and measuring the movements of this plate relative to the base of the footing. Studies were also made of the contact pressure distribution and the effects of density and pressure on the shear strength and compressibility of the Luvia sand; the pressure range used varied from 0.16 psi to 400 psi.

Excellent agreement of the measured settlement with the theoretically calculated settlements was obtained for surface footings on dense sand (relative densities of 78% and 87%), illustrating the validity of the assumed plastic behavior.

For surface footings on loose sand (relative density of 4%), however, the measured settlements did not agree with the theoretical settlements. The reason for this lack of agreement was, in the opinion of Bond, that sand in its loosest state is so compressible that the shear surface pattern which was assumed does not develop. This assumption is undoubtedly valid, and Bond showed that more accurate results are obtained, for low load values, by assuming vertical compression with no lateral deformation.

In spite of the initial assumptions not being correct for loose sands, fair accuracy in prediction of settlements can be obtained from this method by using carefully chosen arbitrary compressibility factors.

The settlements of circular footings of various diameters are compared with the settlement of a 12 inch diameter footing, for the same mean contact pressure, in Figure 4.

B.2.17 Berezantzeu and Yaroshenko (1957)

In these tests performed in the U.S.S.R., still photographs and movies were used to study the character of deformation and stability of dense and loose sands. A description of a subsequent tracing of soil trajectories is also included. No details are given as to test set-ups or shear strengths. The general conclusions of the authors regarding the nature of deformation and influence of size of loaded area, density, and depth are as follows:

Initially, soil particle movement is essentially vertically downward. Increased settlement in both dense and loose sands is accompanied by formation of a compacted core which displaces soil laterally along slip surfaces. These slip surfaces reach the surface in the case of shallow footings on dense sand or terminate in the soil mass in the case of deep foundations on loose soil. Failure is defined in the following ways:

(1) When there is an upheaval of soil and the beginning of general soil displacement along the sliding surfaces can be observed.

(2) When there is no upheaval, and the intense increase in foundation settlement of the sliding zone and the compaction zone interaction can be observed.

Their observations regarding the effects of density and depth on the mode of failure and deformation agree substantially with those of Meyerhof, i.e., "The experimental results confirm that the compressibility of sand increases with increasing size of the foundation (no data given) for the same soil density and depth ratio D_f/B, and a failure with a well defined rupture surface and an upheaval of the ground is observed only with surface and shallow foundations on dense sand."

Theoretical results showing the relation of q_u vs. φ vs. D_f/B are given and show reasonable agreement with the test data given. In the case of deep foundations (D_f/B 3.0) on dense or medium sand and even surface foundations on loose sand, the bearing capacity is likely to be determined only by settlement (i.e., no plastic zones form which are subject to analysis).

B.2.18 Tcheng (1957)

The mode of failure and bearing capacity of a two layered system was studied, i.e., a shallow sand layer resting on a deep clay deposit. The conclusion was that when the thickness of the upper layer is less than 1.5 times the foundation width, the rupture lines in the upper layer are vertical.

B.2.19 Schneebeli (1957)

This paper describing the use of cylindrical plastic rods to represent a two dimensional cohesionless mass and to minimize the influence of the wall on the test results represents an interesting development. Tests in a special triaxial compression apparatus showed that the rods do behave similarly to sands having angles of friction varying from 24 to 35 degrees depending primarily on the surface roughness. The author states that it is necessary to use rods of at least two different sizes to obtain an isotropic medium; thus, Nylon, Dural lisse, and Dural sable, varying in diameters from 2 to 9 mm were used. Pictures of time exposures of retaining walls indicate that a very good detection of deformation can be made.

B.2.20 Hansen (1957)

In his General Report (Section 4a) at the 4th ICSMFE Brunch, Hansen refers to model tests by Bent Hansen on circular rough footings of "dry" sand. Even after corrections for possible cohesion and the scale effects (no elaboration given), the actual values of N_q and N_γ were several times those computed on the basis of friction angles from triaxial tests and conventional formulae.

B.2.21 Sowers (1958)

In a discussion at the 4th ICSMFE, reference was made to loading tests on plates 1 ft. sq. to 3 ft. sq., from sand deposits of the Southeastern United States. The soils were of clean, uniform, angular quartz and had a medium to fine grain size. The soils in situ existed at relative densities of from 30 to 70 percent. Friction angles determined in the laboratory varied from 33 to nearly 40 degrees. Sowers states that the computed bearing capacity (using Terzaghi's general shear analysis) in all cases was approximately 30 percent greater than the measured value, (however he does not state whether or not he made the reduction suggested by Terzaghi for square footings).

B.3 DISCUSSION OF TESTS ON SAND

Although considerable attention has been focused on the factors influencing the bearing capacity of footings on sand, it must be pointed out that studies of this type are somewhat academic. In actual practice, bearing capacity is seldom a consideration (generally only for the very small footings). However, where the investigation has also considered the deformation under load or the mechanism of failure, considerable fundamental knowledge of the stress-strain behavior of soils can be obtained.

B.3.1 Bearing Capacity

Small scale tests on footings have been performed to study bearing capacity and the effects of (1) size and shape of the loaded area, (2) depth of confinement, (3) roughness of the base, and (4) ground water conditions.

When evaluating whether or not a particular theoretical solution gives a satisfactory answer, the only approach which can be used is to compute a required friction angle on the basis of the proposed theory, compare this required friction angle with the angle obtained in the laboratory, and check the reasonableness of the computed friction angle. This is the only rational approach because as several investigations have pointed out, the friction angle is highly dependent on density and pressure. At the low pressures used in model studies this is particularly critical.* Not only is the variation in ϕ with σ generally rapid at low pressures, but, as an illustration (using Meyerhof's data) a change in density from 98.8 to 102.5 lbs/ft^3 at 0.1 tons per sq.ft. will change ϕ from approximately 41-1/2 to 45 degrees. Far more important, however, is the rapid change in bearing capacity with ϕ at high friction angles; for the ϕ variation, mentioned above, the bearing capacity could easily change by 100 percent.

From the data presented in the papers reviewed, average or typical values of the experimentally determined bearing capacity (q_u) were selected. Using these values and Terzaghi's formula (TERZAGHI and PECK 1945) for surface strip footings, i.e., $q_u = 1/2\ B N_\gamma$, a required value of N_γ (without a shape correction) was determined.

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*Some data regarding this effect are presented in Appendix L

With this required N_γ, a corresponding required friction angle ϕ_{req} was obtained from charts prepared by Terzaghi or Meyerhof. The computed values of required N_γ and ϕ_{req} are tabulated in Table II and can be compared with values of the friction angle determined from the results of direct shear or triaxial tests. It can be seen that all of the values of ϕ_{req} represent possible friction angles for sands at different densities and confining pressures.

Pandya's study of shear strength indicated practically no influence of initial density and pressure on the friction angle. His footing test results, however, required a friction angle of about 40 degrees as opposed to the 29.5 to 31.5 degrees he obtained in shear box tests. Because these tests were run in the early 1930's before satisfactory development of shear testing, it is possible that his test results are not satisfactory (this could be easily checked by performing tests on Annisquam sand).

In those cases where the soil was adequately tested, the required friction angles tend to lie within the range of experimental values. Meyerhof claims good agreement between required and lab test friction angles, provided the experimental value is evaluated at a pressure corresponding to approximately 1/10th of the applied pressure (presumably the average normal stress along the failure arc).*

Sylwestrowicz determined only the angle of repose of his soil, but it is interesting to note that this angle agrees well with the required friction angle for the final bearing capacity. Unfortunately this was not the case in Meyerhof's work, and he suggests that the difference is possibly due to the partial and progressive nature of failure along the failure surface.

LUNDGREN and HANSEN (1958) present a chart (p. 220) showing the relationship between N_{emp} and N_q and N_γ in which N_{emp} is larger than N_q or N_γ at friction angles in excess of 30 degrees. It should be mentioned

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*Recent work, not yet published, at Imperial College, University of London, sheds more light on the effect of intermediate principal stress upon the friction angle of sands. It would appear that this effect is very important and must be taken into account in future work.

that the theoretical curve presented does not agree with MEYERHOF'S (1951) for surface footings above 33 degrees. (It may be based on Lundgren's and Mortensen's analysis). It also should be noted that if the highest N_{emp} were plotted at a friction angle 5 degrees higher (45 degrees instead of 40 degrees) reasonably good agreement with Meyerhof's theoretical curve would be obtained. The experimental results all confirm the theoretical conclusion that the bearing capacity is a function of width, although not necessarily a linear function. Meyerhof and others discussed this and attributed the non-linearity to the rapid change in friction angles with pressure; this effect is particularly prominent in small scale footings on dense sands. For normal footings on sands or large footings on dense sand the variation is not as great. The relation between bearing capacity and width is summarized in Figure 5 (from Meyerhof 1950) to which has been added the data of Pandya and Eastwood. It can readily be seen that all the data conforms to the same general pattern; the shapes of the curve represent empirical scaling laws. Variation may be due not only to differences in soil density but also differences in friction between the base of the footing and the soil for different types of loading plates.

The data of Meyerhof, Pandya, and Berezantzev all show the marked increase in bearing capacity with depth, and the experimental results of all three seem to be in reasonable agreement. It is difficult to compare directly the data of Meyerhof and Pandya. Pandya limited his studies to a maximum depth to width ratio of 5:1; Meyerhof studied depth effects to a maximum depth to width ratio of 25:1, but the data as plotted are too crowded in the D/B range, zero to five, to permit interpretation.

Although Meyerhof claimed that a clear distance of 10 times the footing width was needed for strip footings and 5 times the footing width for square or round footings, it appears from all the data received that as long as the clear distance is larger than the extent of the failure surfaces, the results could be considered to give reliable values of bearing capacity.

The results of Meyerhof's studies of the effect of footing shape on bearing capacity indicated that, for surface footings on dense sand, the bearing capcity increases slightly from a circular to a square shape and very rapidly from a square to a short rectangular shape.

Above a length to width ratio of 6:1 the increase is small. For compacted sand the variation is similar but less pronounced, while for loose sand, there is no appreciable difference between the bearing capaicty of circular, square or rectangular footings (at least up to a length to width ratio of 36:1). The available data on the effect of footing shape on bearing capacity has been summarized by Meyerhof (Figure 4). From all of the evidence it was concluded that the effect of shape is a function of the relative density and friction angle of the sand, and the major source of differences in the various investigations is undoubtedly due to the differences in soil density (the effect of shape is particularly noticeable for dense sands and is hardly detectable for loose sands). In addition, it is also possible that in those cases where the length of the footing was almost as large as the width of the bin, there were significant restraining effects due to the pressure of the bin. In connection with this factor, the use of cylindrical rods, laid horizontally on top of one another, to represent a 2-dimensional mass appears to the writer to have considerable merit.

The effects of footing roughness and ground water conditions have been evaluated by models and are in agreement with theoretical ideas.

B.3.2 Settlement

In connection with scaling laws for the settlement of footings on sand, there is still considerable difference of opinion. Gilboy and Pandya concluded from their studies that the settlement under a given pressure (at low pressures compared to the ultimate) was essentially independent of size. However, the data of Kogler (Figures 1 and 4), Press (Figure 7), and Bond (Figure 4) suggest that settlement at constant pressure is indeed a function of size. JANBU, BJERRUM, and KJAERNALI (1956), using available literature and the results in settlement records in their institute as a basis, have suggested an even greater size effect (especially for loose soils) than had hitherto been considered (Figure 8).

The results of load tests conducted in connection with pavement studies show an effect of size on the coefficient of settlement ($K = \frac{\rho}{\delta}$) up to a diameter of about 20 to 30 inches; above this size the diameter appears to have little effect. In the field studies which have been conducted (PHILIPPE 1948), (Figure 9), it is quite possible that the effects are also a function of the sub-base and subgrade characteristics since the data from Philippe mention only that the supporting medium was a uniformly compacted 18 inch sand and gravel base. With no data given on the subgrade, the general validity of the data's indication of size effects is open to question.

At the present time, scaling laws for predicting settlement of footings on sand are still inadequately understood and the empirical data are such that no general conclusions can be drawn. About all that can be done at the present time is to set upper and lower limits based on the information shown in Figures 7, 8, and 9.

The difference in distribution of contact pressure as a function of size has not been considred but could conceivably play a role in explaining the size effects. For sands, the pressure would vary, roughly, from parabolic for the small footings to almost uniform for the larger footings.

In addition, ROWE (1959) shows, from theoretical considerations which are in reasonable agreement with limited experimental data, that in the model range 0-2 feet for loose sand and 0-5 feet for dense sand, the lateral at-rest coefficient of pressure, K_o, varies rapidly with depth. This in itself may have an important influence on the extrapolation of deformation studies on models to field behavior.

B.4 SUMMARY OF SMALL SCALE TESTS ON CLAY

Only six references have been found in which studies of the behavior of footings on clay are reported.

B.4.1 Golder (1941)

Strip footings (3 in. x 18 in.) and square footings (3 in. x 3 in.) were tested on a remolded London clay which was packed tightly into a wooden box (2' 6" x 1' 6" x 8"). After each test the clay was sampled for shear tests and moisture content determinations. The mean bearing capacity for the strip footings was 5.1 times the shear strength, and the mean pressure at failure on the square footings was approximately 30 percent higher than that of the strip.

B.4.2 Skempton (1942)

An investigation of a building footing, 8' 0" x 9' 0", founded at a depth of 5-1/2 feet below ground level was made. By backfiguring from building loads, the ultimate bearing capacity was estimated to be 2500 lbs. per sq.ft. (net pressure). Tests on undistrubed samples indicated undrained shear strengths of about 350 lbs. per sq.ft. for the clay below the footing and above the foundation level, about 550 lbs. per sq.ft. For the case cited, 6.7 times the shear strength (based on Golder's results) equals 2550 lbs. per sq.ft. (neglecting the strength of soil above the foundation) which was within six percent of the actual failure stress.

B.4.3 Osterberg (1948)

Six sets of data for bearing tests on plates, varying in size from 4 inches to 84 inches, were reported. The tests were primarily on cohesive soils (silty loam, silty clay, clay shale, sand and clay), and one series of tests was on 18 inches of sand and gravel compacted over the subgrade. Osterberg presents plots of unit stress versus the ratio of settlement to plate diameter (both to logarithmic scales). In general, the data fall on a reasonably unique line which appears to be independent of plate size (in three instances, the uniqueness of the line is remarkable). The lines tend to be straight in the range of low pressures and low deflections to diameter ratios. At higher values of each parameter, the curve tends to flatten out (possibly tending to reach an asymptotic pressure corresponding to the bearing capacity). The method of plotting used by Osterberg allows direct observation of any size effects and makes it possible to predict settlement of any size and shape footing. In addition, the results of laboratory triaxial and unconfined compression tests and one-dimensional consolidation tests may be plotted for comparison. In one illustration presented, the use of laboratory test data results in an overestimate of the settlement (by as much as a factor of 5 if the estimate is based on the results of the unconfined compression test).

Observations of a footing, 40 ft. by 50 ft., are plotted for the Panama Canal Cucaracha Formation and can be compared with the results of test on rigid plates 8 inches, 16 inches and 40 inches in diameter. Up to a pressure of 20 psi the agreement is good; at a pressure of 100 psi the plate values are about 50 to 100 percent larger than the value for the large footing. This result is probably due to the difference in average confining pressure in the different cases; the difference in average confining pressures would be slight for the plates 8 to 40 inches in diameter but substantially larger for the footing 40 ft. by 50 ft.

B.4.4 Skempton (1951); Meyerhof (1951)

Both authors in separate papers refer to model footing studies by Meigh at Imperial College who ran footing tests on undisturbed and remolded clays. These test results, plus available field data, are used to evaluate Meyerhof's theoretical solution for bearing capacity and are used to establish semi-empirical design recommendations for any shape footing at any depth in cohesive soil.

Meyerhof discusses the mode of failure in the model tests and points out that in the case of stiff clay, a well defined failure (for surface footings at least) similar to that assumed in the theory, develops. For the stiff clay there is good agreement between experimental and theoretical values of bearing capacity for strip and circular footings at various depths. The bearing capacity of a square footing is a little smaller than that of a circular area and both are about 20 percent greater than that of a strip footing.

In the case of soft clay, on the other hand, local shear failure occurs without the development of a noticeable rupture surface, and the bearing capacity is not well defined except for deep foundations. Because of the local shear failure in soft clay, the greater penetration required for development of shear strength gives a bearing capacity greater than the theoretical for a shallow footing and smaller than the theoretical for deep foundations.

Skempton pointed out the necessity of making corrections for consolidation during loading and for different rates of strain during the unconfined tests (from which shear strength is evaluated) and the model tests. In addition, Skempton indicated that when the load settlement curves were plotted in a dimensionless form (q/c vs $\frac{\text{Settlement}}{B}$) they were about the same for all footings.

B.4.5 Rocha and Folque (1953)

Two cases are described where model studies were used to estimate the settlement of full scale structures. In both cases, a length scale equal to 1/100 of the prototype was used; this, according to the authors, resulted in a time scale for the model equal to 1/100,000. It should be noted, however, that the text suggests a time scale equal to only 1/100. The authors state that the soil (sandy clay and clay) for the model studies was obtained from undisturbed samples (no details). The dead weight of soil above the foundation was reproduced by applying a superimposed stress to the surface of the soil in the model test. The results presented by the authors indicated reasonably good agreement between model and prototype behavior; the difference, the authors suggest, is possibly due to secondary time effects in the prototype which are not present in the model. In the case cited, the settlement prediction based on models is considerably better than the predictions based on results of consolidation tests on undisturbed samples.

B.4.6 Bergfelt (1956)

The results of loading tests on plates of various sizes and shapes (3 cm round, 30 cm square, 15 x 60 cm, 30 x 120 cm, and a few combination square and circular plates) agreed well with the results presented by Skempton and Meyerhof regarding the relation between failure load and shear strength and the relationship between bearing capacity and shape of loaded area.

In addition, deformation within the clay mass was studied using X-ray pictures to detect the movement of lead shot inserted into a block of undisturbed clay (35 x 35 x 35 cm). Fairly good agreement between the largest observed displacements for a square plate (7 x 7 cm) and the Steinbreuer curve was noted. In the model tests no slip was observed even though settlement reached 10 percent of the plate width.

B.5 DISCUSSION OF TESTS ON CLAY

The few reported investigations of small tests on clay have generally substantiated theoretical approaches to bearing capacity (at least for the stiff soils). Results to date have also established quite reliably the effect of the shape of the loaded area.

The data also point up, however, the necessity of empirically considering the stress-strain characteristics of the soil. When the stress-strain relationships closely approximate those assumed in plastic behavior, then the net results are reasonably good. However, the tacit assumption that strength is independent of pressure is not valid, and undoubtedly a different failure surface develops than that assumed by the theory of plasticity. In the case of soft soils (remolded), bearing capacity is more dependent upon settlement; the stress-strain characteristics are such that soil cannot transfer stresses outward laterally, and so-called local shear failure occurs.

B.6 GENERAL COMMENTS REGARDING SMALL SCALE FOOTING TESTS

To date, the ability to predict footing behavior has developed from either a purely empirical approach wherein deformation is determined as a function of size and shape, the results are plotted, and empirical relations are established, or from a theoretical approach the validity of which is evaluated from the results of tests at reduced dimensions. This second approach has been satisfactory in the computations of bearing capacity to the extent that the friction angles required by the theoretical solution usually can be considered as a possible value for the soil being used.

There has been very little study of footing behavior from the point of view of dimensional analysis or from a study of the stress-strain characteristics of the soil.

ROCHA (1957) attempts to point out the properties which materials used to represent soil in model studies must have in order to have proportionality of stresses, strains, displacements, and time between the model and prototype. According to Rocha, when the weight of the soil can be neglected, the prototype material can be used and conversely "...when the weight of the mass itself is taken into account the materials of the prototype cannot be used for the construction of the models." The use of plastics and plaster of Paris for the structural part of the model is explained in Rocha's report, but he does not give any illustrations of how this has been or could be effected. Only certain relationships which must exist between the soil of the model and prototype, and the fact that the choice of materials demands triaxial tests, are indicated.

This review has made it apparent that in any study of the behavior of sand under load, it will be necessary to consider separately the dense, the medium, and the loose condition. Dense soils, because of their stress-strain and volume change characteristics, tend to be susceptible to study because of the so-called theories of plasticity. Medium and loose sands, however, because of the volume compression which occurs during deformation, generally cannot develop modes of deformation assumed in theory. Different conclusions regarding footing behavior have been reached by different investigators, and it is quite possible, as Meyerhof points out, that the variation is due to the different relative densities being used. Similar remarks are applicable to studies on clay.

These remarks point up the need for more fundamental study of the stress-strain and volume change characteristics of soil. When these characteristics can be put into more specific terms, then it may be possible to at least formulate dimensional relationships. At present, the factors affecting the soil behavior are far from being totally understood. As an illustration of one factor which is just beginning to be studied, recent unpublished research at Imperial College indicates higher strengths and smaller strains at failure for a sand which was sheared in essentially a plane strain condition compared to the same sand sheared under conditions of equal lateral pressures in all directions (standard, cylindrical compression). This discovery in itself could possibly account for the effect of shape on the bearing capacity of dense sands.

In any future model studies, the use of either time photography or X-ray techniques to study the deformation of soil masses should be considered. In addition, the use of a stack of small diameter cylindrical rods to approximate a soil mass warrants consideration; use of these rods has two advantages: (a) it is easier to observe particle motion and (b) end effects can be minimized.

To date, no study of the effect of particle size in relation to footing size appears to have been made. There is usually a general discussion about the fact that in models the structural part is scaled down, but the soil is never scaled. Although Rocha has pointed out the relationship which the model must have to the prototype when the deformation is due to gravity forces, the solution to the problem of obtaining satisfactory model material to represent soil is still in the embryo stage. When the mode of rupture is being studied to verify a theoretical approach, it is satisfactory to treat the soil as a continuum as long as the effect of low confining pressures on the friction angle and volume changes is recognized. However, Figure 10 shows how stress is likely to be transmitted throughout the mass of a granular medium, and it is possible that particle size can have some effect on scaling laws. This area deserves study.

It is desirable in any model studies, to use a granular material in which properties are scarcely affected by changes in void ratio and pressure. Possibly uniform spherical glass beads could be used.

As well illustrated by the results of LANDALE (1954), very careful control of moisture is needed; a small amount of moisture can impart to the soil an apparent cohesion which will make the soil appear to behave as a cohesive rather than a cohesionless material.

According to Meyerhof, it is necessary to have a bin considerably larger than the extent of the potential failure surface because movement can be detected at distances 2 to 3 times the limit of the failure surface. However, other studies, in which it was considered important only to have the bin larger than the failure surface, seem to give reasonable results. More study is needed to determine the precise effects of having the edges of the bin beyond the failure surface or beyond the point of no detectable movement.

The roughness of the contact surface has also been shown to have a marked effect on the results of model studies. The approach of ROWE (1954) to defoDmation problems appears promising, at least in the case of the dense sands.

As an aid to interpreting the results of model studies, the data should be plotted in terms of dimensionless parameters. This use of dimensionless parameters has met with reasonable success in Danish research on footings and piles of sand (LUNDGREN 1956), and the success of such an approach by SKEMPTON (1951) and OSTERBERG (1958) has already been mentioned.

B.7 CONCLUSIONS

Scrutiny of these various experimental programs has led to the following conclusions concerning the usefulness of small-scale testing techniques.

(a) The greatest success is achieved when the objective is a non-quantitiative verification of assumptions through study of the modes of deformation or patterns of behavior. Examples of successful small-scale footing experiments are: those intended to verify the existence of rupture zones and to determine their general extent.

(b) Good results are achieved when small-scale experiments are designed to supplement established theories, by providing empirical correction factors to describe effects that cannot be evaluated by theoretical calculation. The work of Meyerhof provides an excellent example. His tests were conducted with great care and patience. The reliability of the model tests was first verified by comparing results for shallow long-strip footings with the theoretical results for this case. Even though experimental uncertainties made it impossible to obtain a close numerical check, the general agreement of experiment and theory was clear. Thus Meyerhof could proceed to use the experiments for determining correction factors for the length and depth of the loaded area.

(c) Every attempt to use small-scale experiments to verify quantitative relationships or to establish numerical values for bearing capacity led to general frustration. It has been apparent at every turn that a fantastic number of details in experimental technique affect to a high degree the absolute quantitative result (as contrasted to relative quantitative result between several tests).

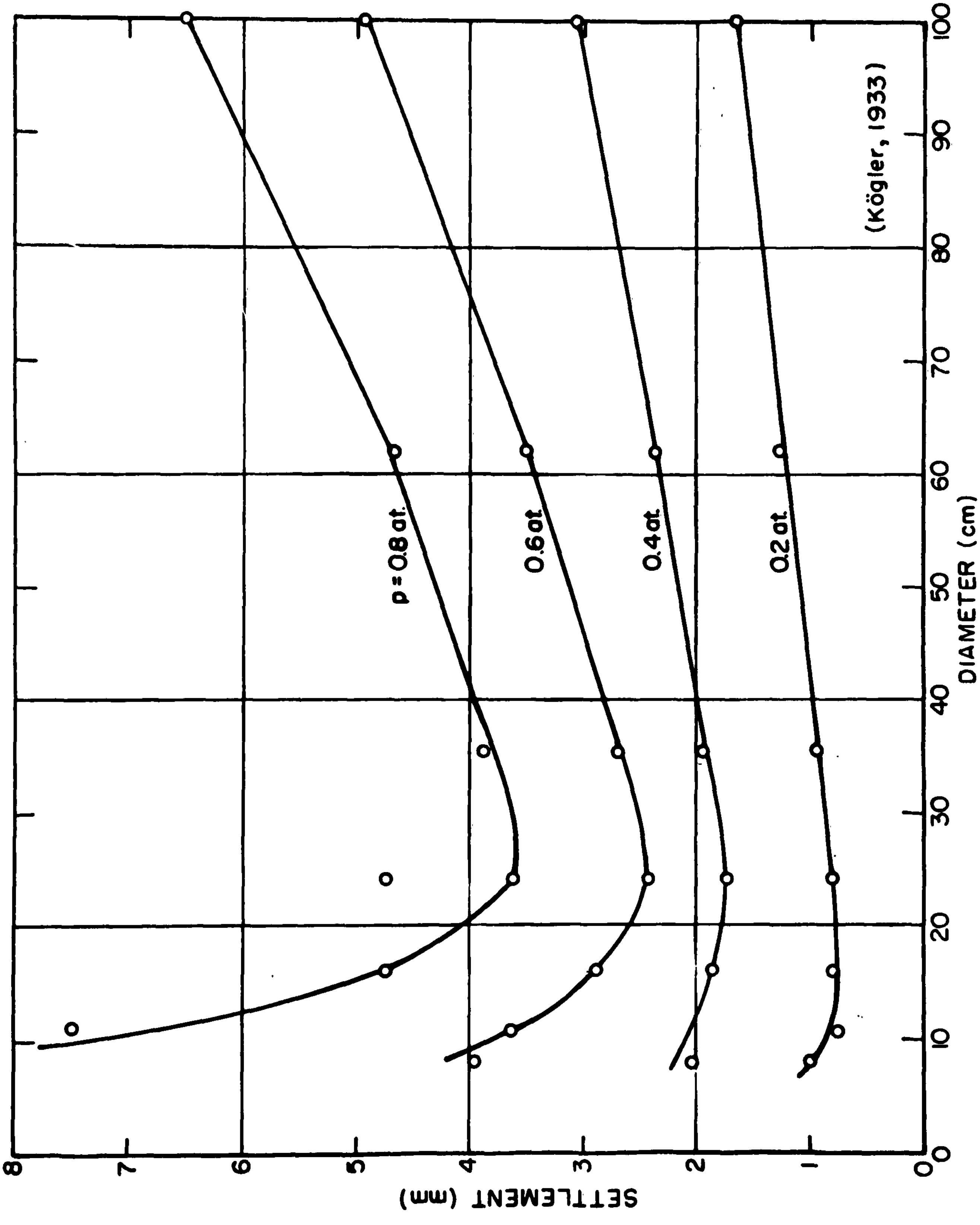
As a final and very tentative conclusion, it seems doubtful that much success will be achieved by attempting to scale soil through use of other materials.

TABLE I
SMALL SCALE FOOTING TESTS ON SAND
GENERAL DATA

REFERENCE	TYPE	d = depth of sand in bin BIN SIZE d w d	MATERIAL	LOADING AREA SHAPE	SIZE RANGE	RANGE OF DEPTHS USED	SAND GRAIN SIZE DISTRIBUTION D_{90}	D_{60}	D_{10}	METHOD OF ALL PLACEMENT	TECHNIQUE OF DENSITY DETERMINATION
Fellenius (1930)	wood with 2 glass sides	100cm x 50 x 60		□ ○ ▭	5 x 5 to 25 x 25 cm 20, 30 to 5 x 60 to 20 x 60	zero	Fine Dry Sand			No data	No data
Jandris & Riley (1930)	wooden box	18" x 18" x 9"	$\frac{1}{4}$" brass	□ ○ & octag.	A = 64 cm^2	zero	Plum Isand Sand same as Gilboy			Compacted to 3 different void ratios	A.erage value
Gilboy (1930)	concrete bin	14' x 14' x 7'	Rei concrete	octag.	A = 1, 3 & 6 sq. ft	zero	1.7 mm	1.0	.51	Compacted to 3 different densities	Sampling with 3" ϕ x 5" long tubes after test
Pandya (1933)	wooden	3' x 3' x 3'		○ □ ▭	4.76 cm 7.6 - 15.2 cm 6.8 - 82.2	0 - 5.3d	2.4mm	.25	.15	tamped in 1" layers e = 0.66	Average
Golder (1941)	wooden	3'-6" x 3'-6" x 14"	Steel filled with concrete	□ ▭	3" ϕ 6" ℓ = 6-30"	zero	In Bin 2.0 Shear Test .8	.5 .48	.2 .23	Tamper w/ 3" □ & attached to electric hammer γ_{mean} = 110.5 (e = 0.51) γ_{range} = 108 - 112	2 cylindrical metal containers open at top, put in and dug out later
Meyerhof (1948-1955)	stiffened steel tank galvanized tank	18" x 15" x 18" 30" x 20" x 24" for water studies	Brass	b ▭ ℓ ○	b: 1/2" - 1" ℓ: 1/2" - 6"		0.6	50% 0.4	0.3	Dense to Medium n = 37% Loose n = 45% placed in 3" layers tamped with vibrating hammer (for modulous)	3" ϕ x 1 1/2 open tins buried at different depths
Davis & Woodward		1 1/2' to 3' d 78 x footing width		○ b ▭ ℓ	2" ○ b: 1 - 1 1/2" ℓ: 10 24"	0-1"	Sandy Soil			Compacted in 1" layers	No data
Eastwood (1951)	wooden box	4' x 3'-6" x 16"		b ▭ ℓ ○ & □	b: 3, 4 & 6" ℓ: 6" - 30" 2" - 12"		1.5mm	.48	0.15	Compacted with hammer	Average γ = 108 open tin cans were dug out after test γ_{range} = 106 - 110
Brebner & Wright (1953)		6' x 2 1/2' x	Steel	b ▭ ℓ	b: 1.87" to 14.8 ℓ: 24"		1.0	0.5	0.15	Compacted	Apparently no control
Sylwestrowicz (1953)	1/2" glass walls	15" x 6" x 10"			b = 2" ℓ = 6"		White sand blast sand			Tamped with steel rod	Average = 103 pcf
Landale (1954)	concrete bin	5' x 5' x 4'		□	3" & 6"		1.4		0.14 0.28	Tamped at W = 3 1/2 - 6%	Loose e = 0.75 Dense e = 0.64 Average
Jumikus (1956)	glass walls	30" x 65" x 12"			4" x 6" x 6"		.35	.22	.09		
Bond (1956)	steel tank	4' x 4' x 8'		○	3 1/2" - 12"	0 - 48"		14	52		2" Static Penetrometer previously calibrated Penetration resistance vs γ_d
Berezantzeu (1957)	Box with glass walls			Strip ○	b: 6cm - 181 d: 4.3 - 60cm	0 to D/B - 4+	No data			No data	No data
Tcheng (195	glass wall	80 x 10 x 40cm		b ▭ ℓ	b: 2.6 - 5.8cm ℓ: 10cm (?)	surface	Spheres of 100μ mean diameter				

TABLE II
COMPARISON OF EXPERIMENT AND THEORY

REFERENCE	TYPE OF SHEAR TEST	DENSITY		SHAPE OF FOOTING	N_γ REQUIRED	ϕ_{reg} (TERZAGHI or MEYERHOF)	EXPERIMENTAL ϕ	
		γ	e				MAX.	ULT.
Fellenius (1930)	No test data			□ & ▭	95 (γ = 100) 79 (γ = 120)	39 - 40 38	31° (estimated)	
Jandis & Riley ( )	By J.E.Roberts(1953) Direct Shear	Dense		○	112 (= 104)	≈ 40	40.7° at 35.8 at	
Gilboy (1930)	"	Medium Dense	0.74 0.69	▭	78.5 (γ = 95) 81A 65 (γ = 89)	38° 37°		
Pandya (1931, 1933)	Direct Shear	Loose Dense	.86 .66		109 (γ = 100)	≈ 40°	29.5° 31.5°	
Golder (1941)	Direct Shear	Dense Loose	e_i = .56 e_i = .75	□ or ▭	189 (γ = 110)	≈ 43°	39° - 48° 32° - 34°	
Meyerhof (1948 - '55)			n = 36% n = 40%		(γ = 104) ← max. 166 ← ult (final)	≈ 46° ← max. ≈ 42° ← final	44 - 49 37 - 41	30.5
Davis & Woodward	Direct Shear & Triaxial	Dense Loose	up to 36° → as low as 15° →	range for 1" footing	(γ = 102) 470 - 680	≈ 47° ≈ 49°		
Eastwood ( )	Direct Shear	110			3" 98 4" 87 6" 82	39 - 40° 37°	43 3/4°	
Sylwestrowicz ( )	Angle of repose	90 #/ft²			302 max. 520 86 final	> 40 > 45° 37°		



SETTLEMENT (mm)
DIAMETER (cm)
p = 0.8 at.
0.6 at.
0.4 at.
0.2 at.
(Kögler, 1933)
0
1
2
3
4
5
6
7
8
0
10
20
30
40
50
60
70
80
90
100

FIG. I

BEARING CAPACITY vs DIAMETER
(SURFACE FOOTINGS)

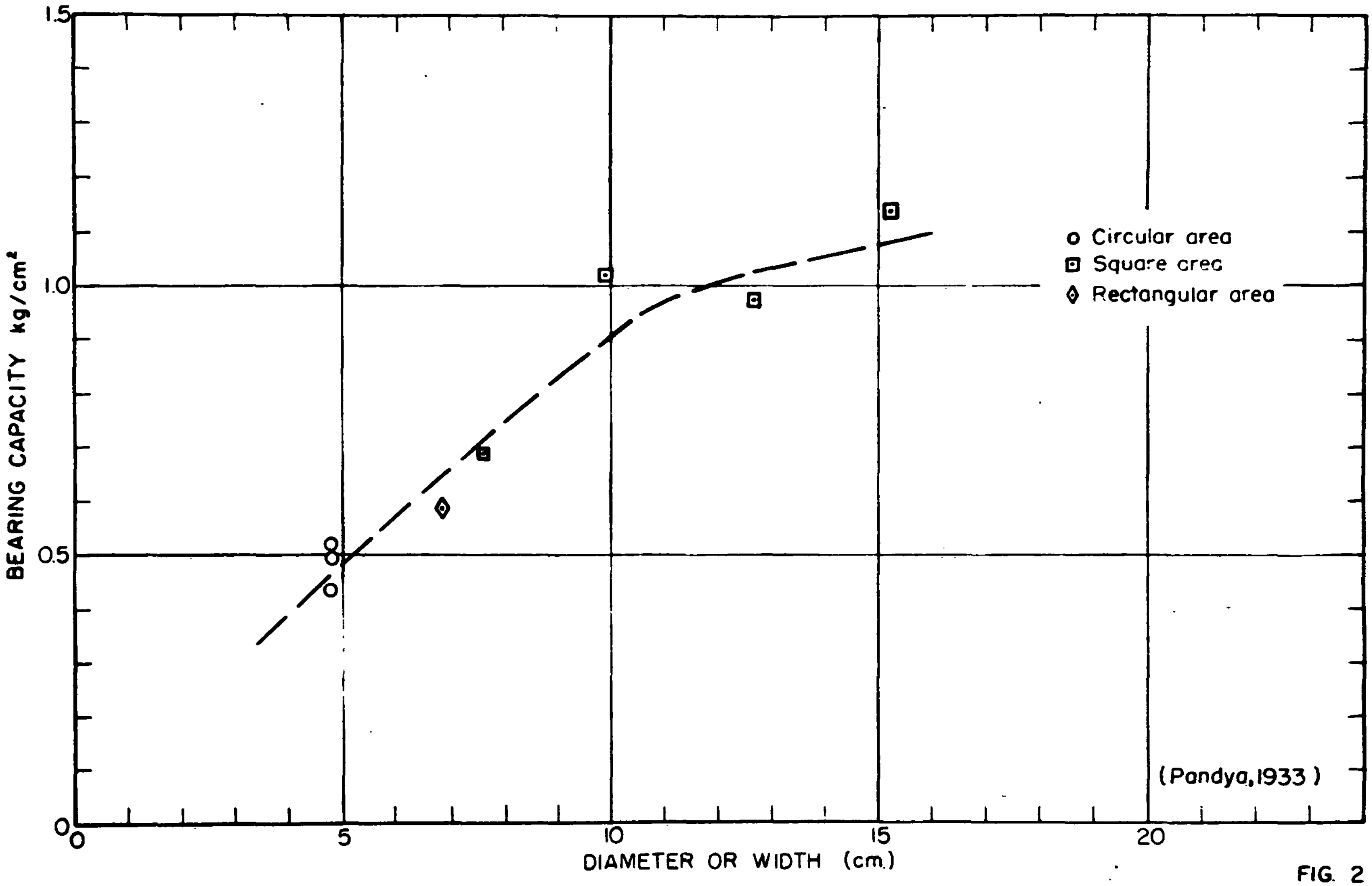
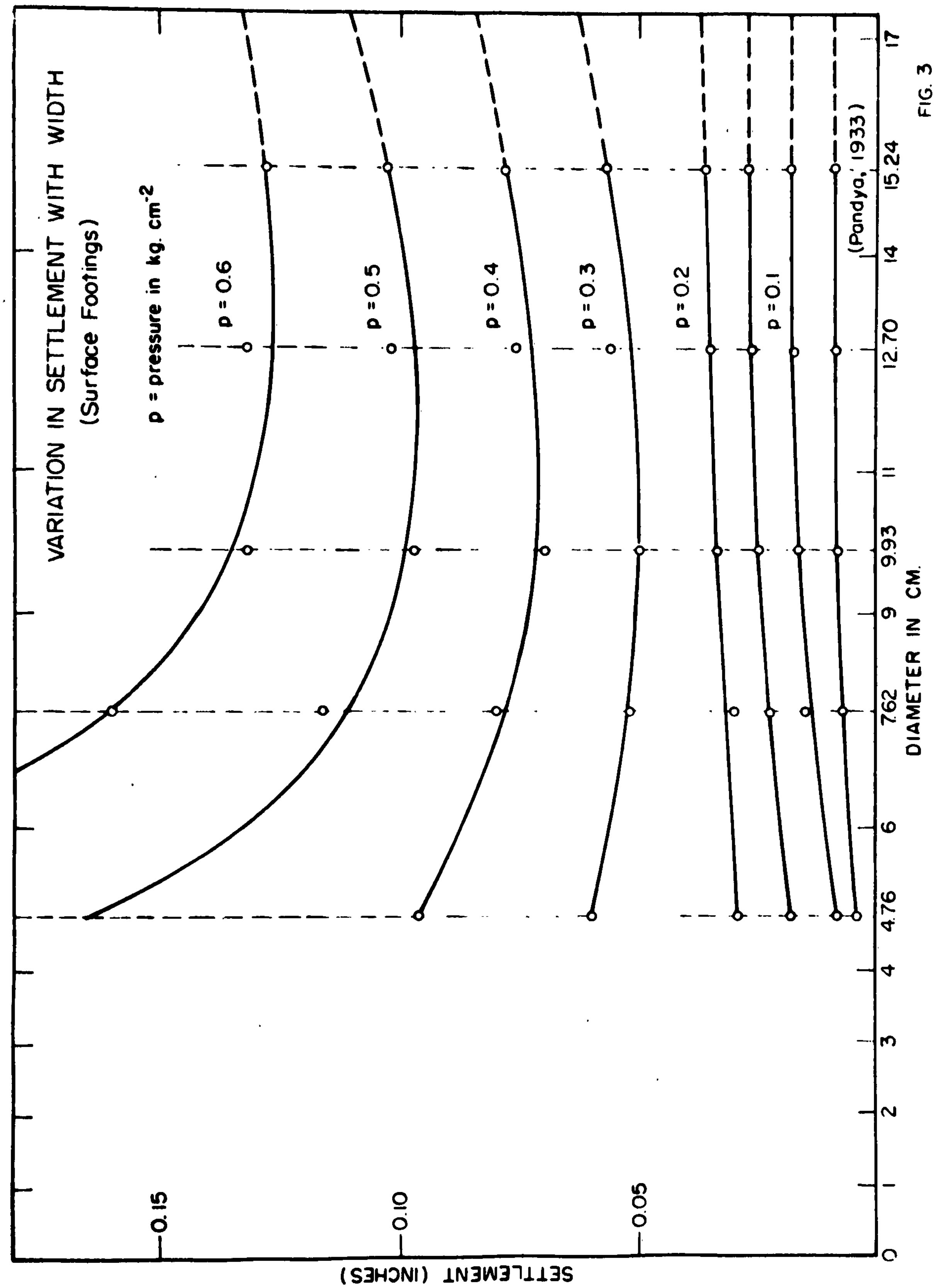
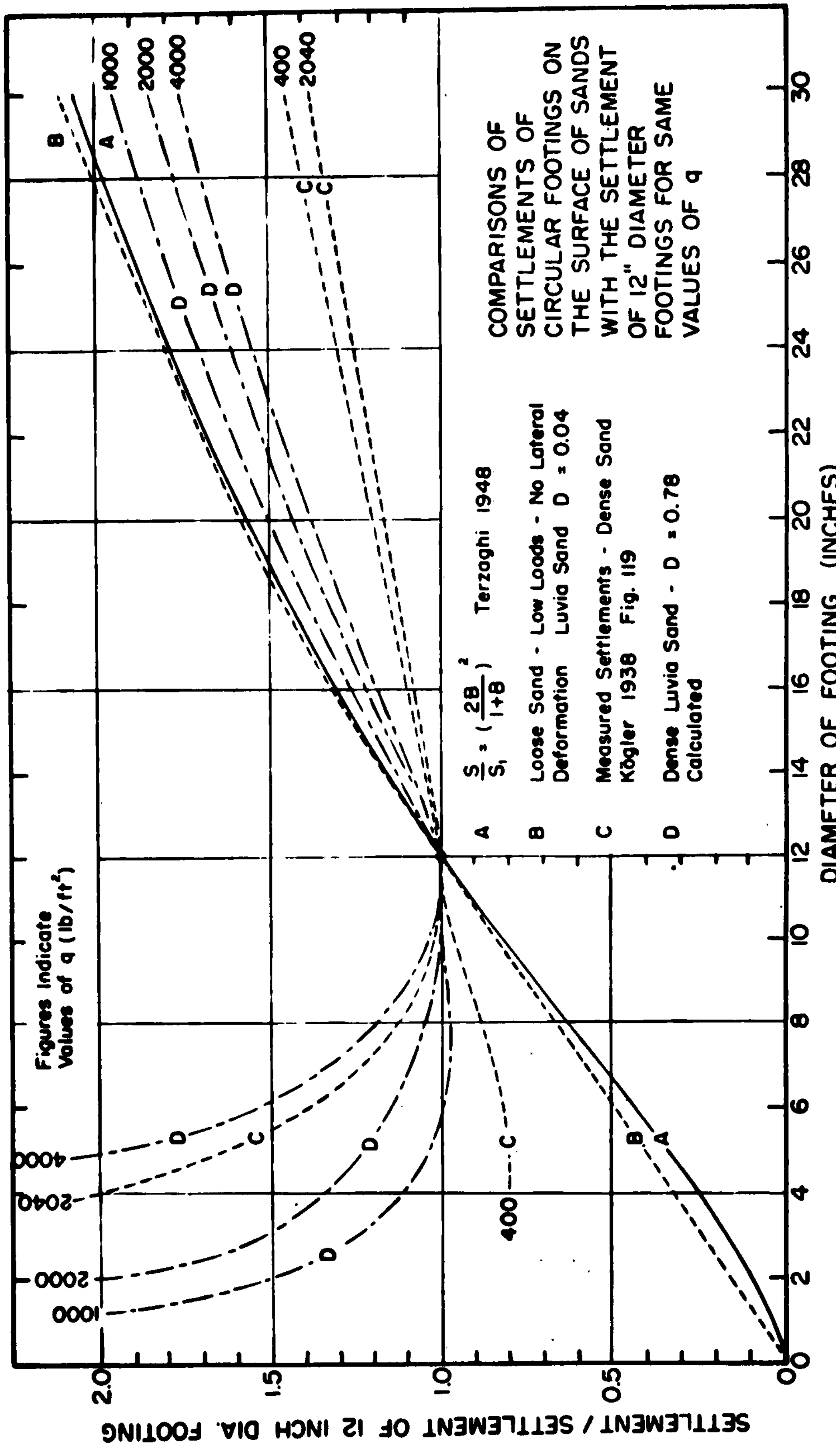


FIG. 2


VARIATION IN SETTLEMENT WITH WIDTH
(Surface Footings)
p = pressure in kg. cm^-2
p = 0.6
p = 0.5
p = 0.4
p = 0.3
p = 0.2
p = 0.1
(Pandya, 1933)
SETTLEMENT (INCHES)
0.15
0.10
0.05
DIAMETER IN CM.
0
1
2
3
4
4.76
6
7.62
9
9.93
11
12.70
14
15.24
17
FIG. 3



(Bond, 1956)

FIG. 4

RELATION BETWEEN BEARING CAPACITY AND WIDTH OF SURFACE FOOTINGS

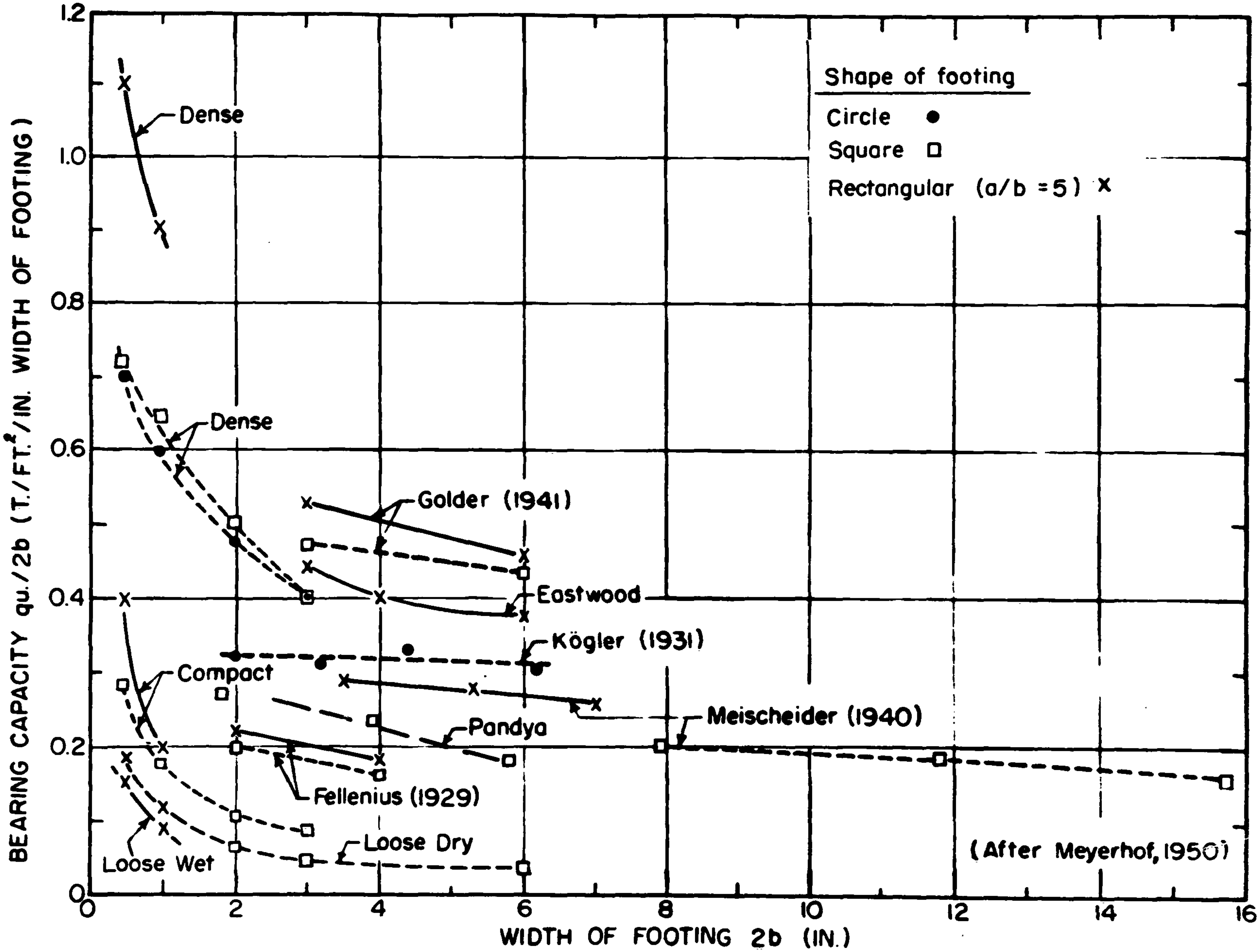


FIG. 5

RELATION BETWEEN BEARING CAPACITY AND SHAPE OF SURFACE FOOTINGS

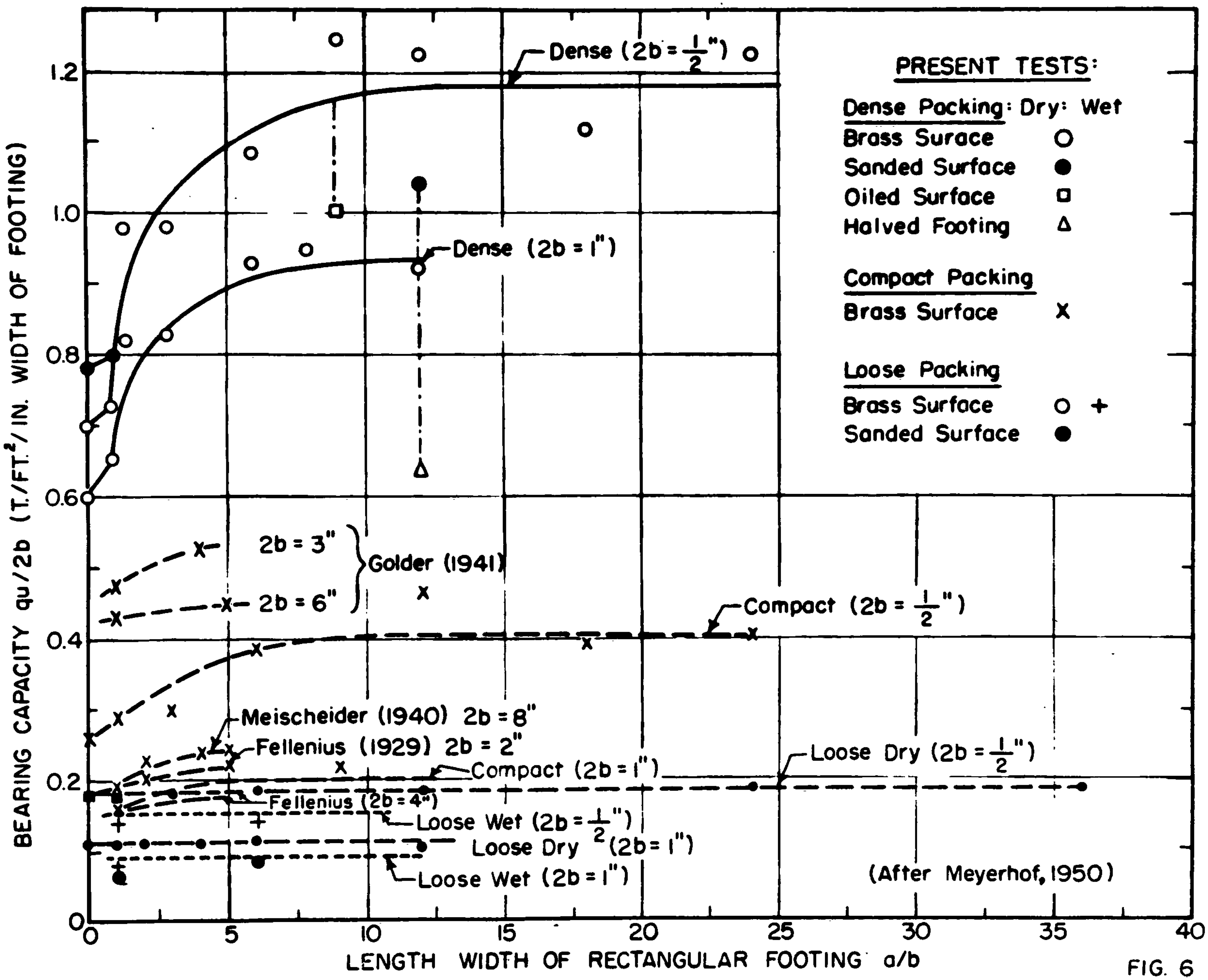
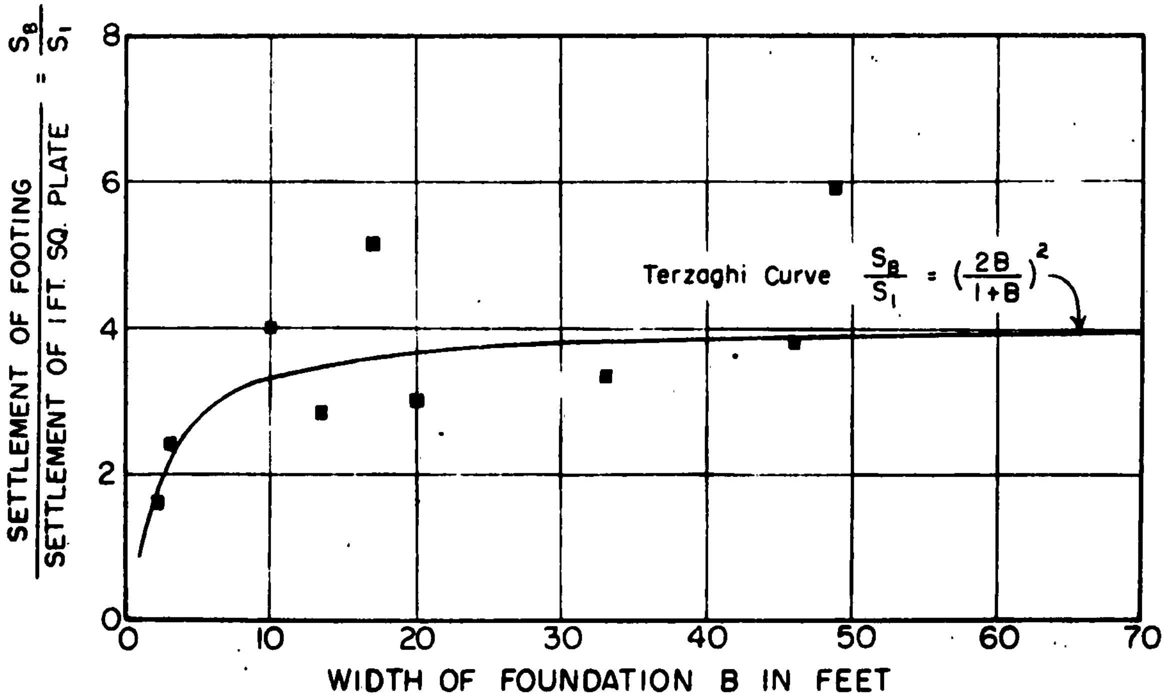


FIG. 6

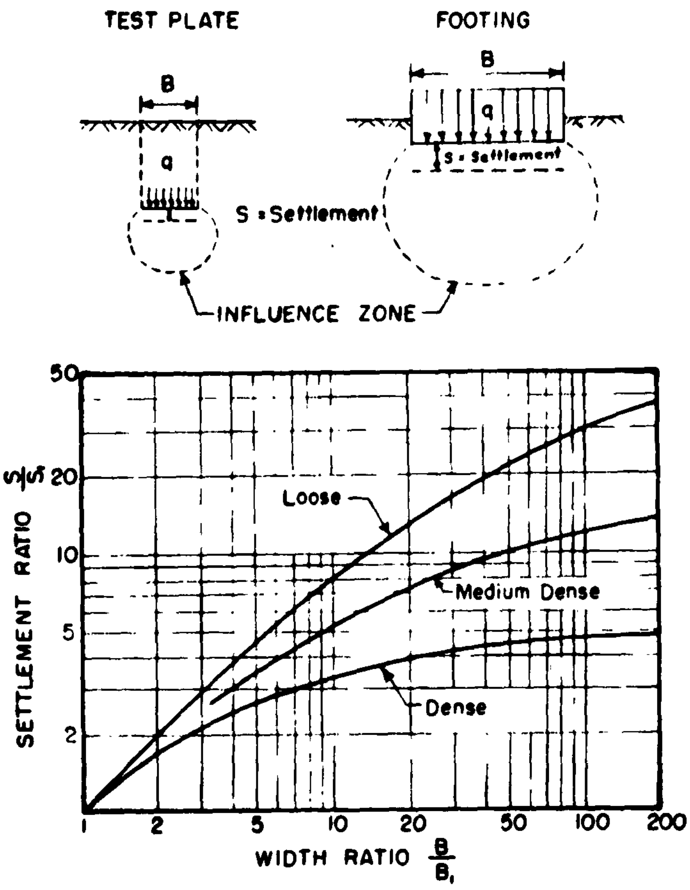


Note: Points shown "■" are from observations
by H. Press, 1932

RELATION BETWEEN SETTLEMENT AND WIDTH OF SURFACE FOOTING ON SAND

FIG. 7

EMPERICAL DETERMINATION OF FOOTING SETTLEMENT ON SAND FROM PLATE LOADING TEST



(From Janbu, Bjerrum and Kjernsli, 1956)

FIG. 8

RELATION BETWEEN SHAPE FACTOR & SHAPE OF SURFACE FOOTINGS

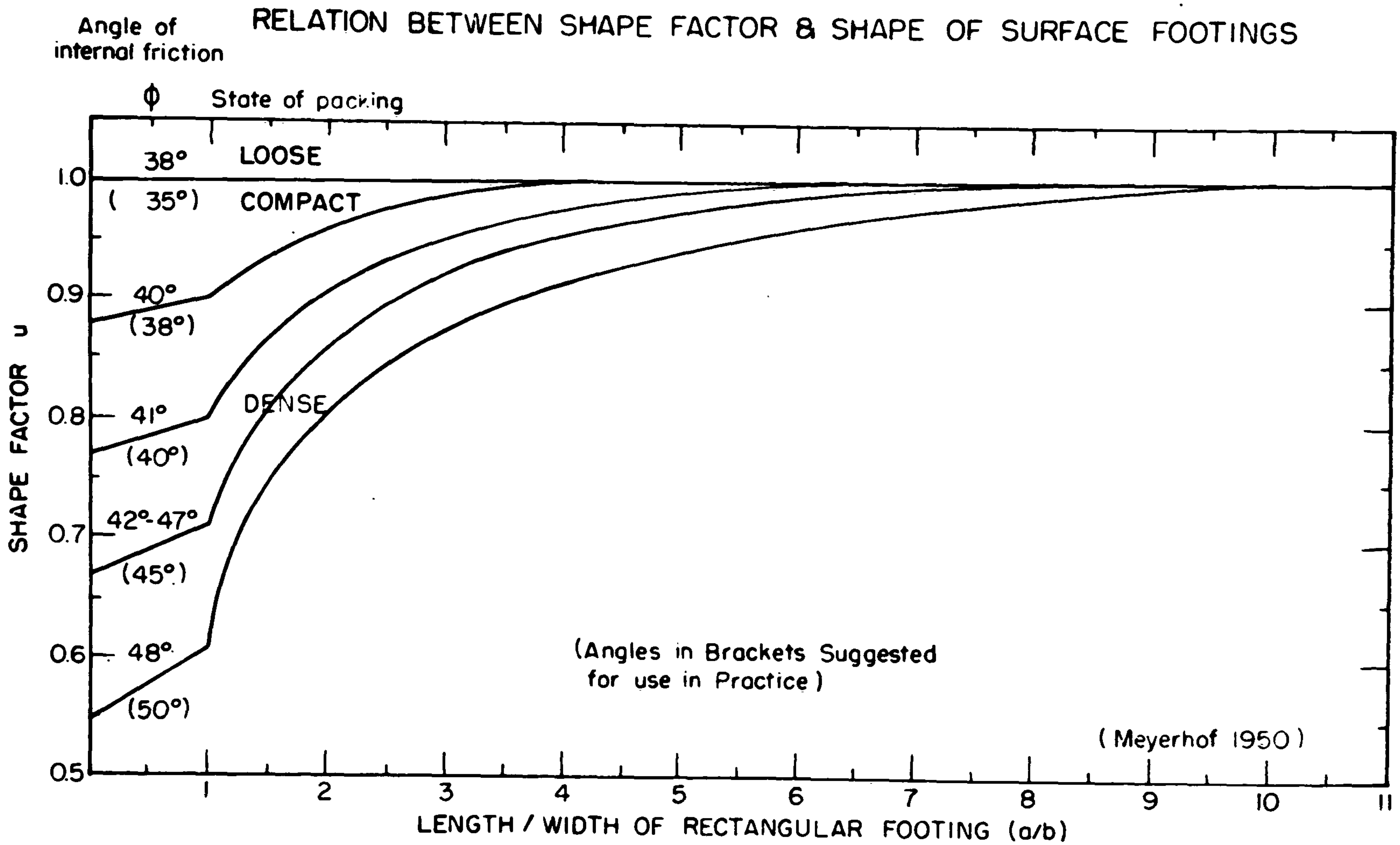
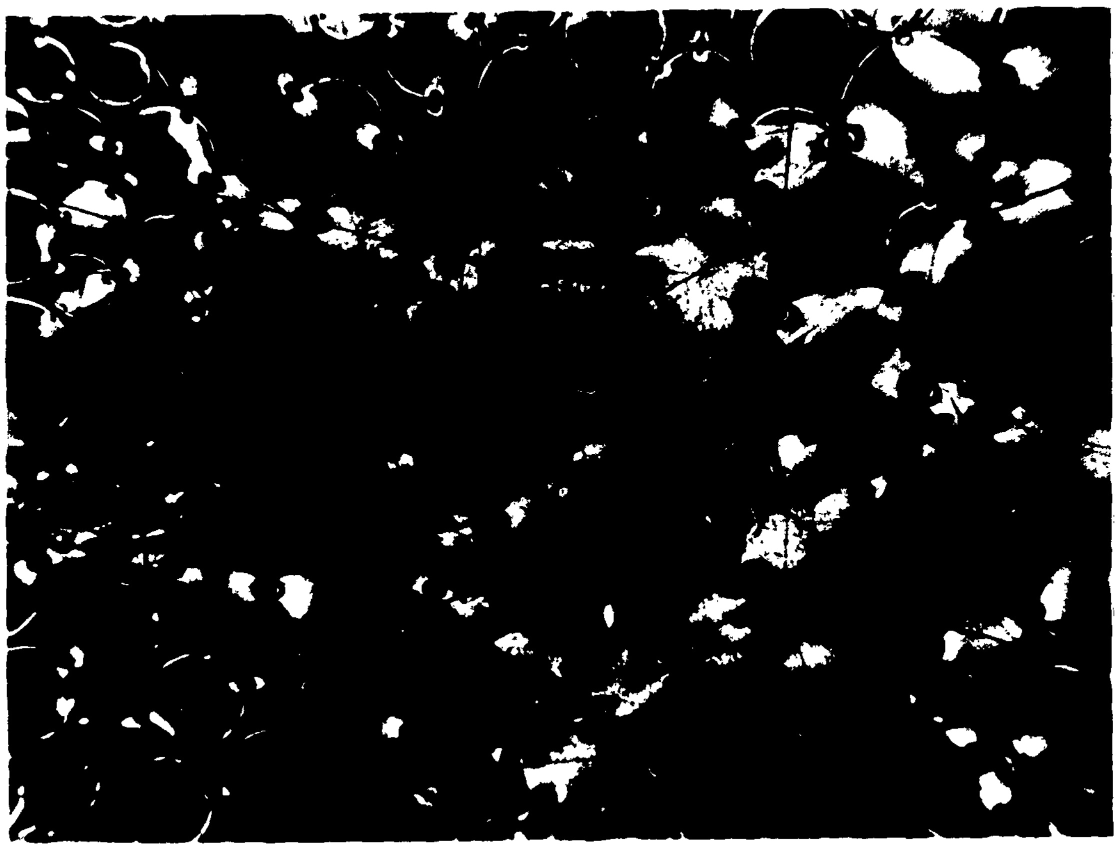


FIG. 9

PHOTO ELASTIC STUDY OF
STRESS TRANSMISSION BETWEEN PARTICLES



From unpublished work of Taylor

FIG. 10

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LIST OF SYMBOLS

a	A distance
B	Width of footing
c	Cohesion per unit of area
D	Diameter of soil sample
D_f	Depth factor
e	Void ratio
k	Coefficient of permeability
w	Water content
N_c, N_γ, N_q	Bearing capacity factors (coefficients in Terzaghi's equation)
N_{emp}	Measured bearing capacity factor
	Total normal stress
σ_1	Total major principal stress, Initial pressure
σ_2	Total intermediate principal stress
σ_3	Total minor principal stress, Confining pressure
q	Intensity of loading under footing
q_u	Ultimate bearing capacity
Δ	1) Deformation 2) Maximum value of wall friction angle
δ	Intermediate value of wall friction angle
Φ	Maximum value of the angle of friction
φ	Apparent friction angle
ρ	Settlement
K	Coefficient of Settlement